Herramientas matemáticas para modelos en dinámica social: desde el estudio del crimen hasta la evacuación de multitudes

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Outline

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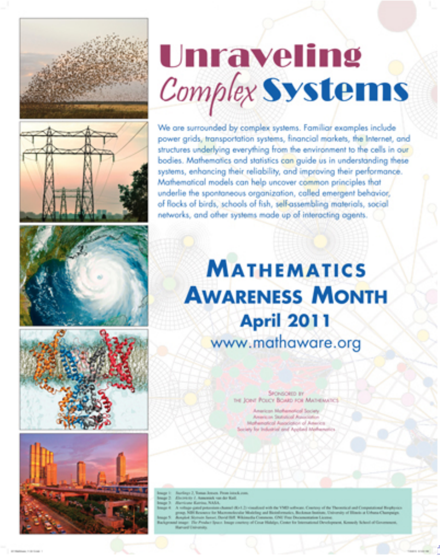
A brief historical excursus on complex living systems

- **Kant (1790)**, Critique de la raison pure, Traduction Francaise, Press Univ. de France, 1967
  Living systems: Special structures organized and with the ability to chase a purpose.

- **Hartwell - Nobel Laureate 2001, Nature 1999**
  • Biological systems are very different from the physical or chemical systems analyzed by statistical mechanics or hydrodynamics. Statistical mechanics typically deals with systems containing many copies of a few interacting components, whereas cells contain from millions to a few copies of each of thousands of different components, each with very specific interactions.
  • Although living systems obey the laws of physics and chemistry, the notion of function or purpose differentiate biology from other natural sciences. Organisms exist to reproduce, whereas, outside religious belief rocks and stars have no purpose. Selection for function has produced the living cell, with a unique set of properties which distinguish it from inanimate systems of interacting molecules. Cells exist far from thermal equilibrium by harvesting energy from their environment.

- **From www.mathaware.org**



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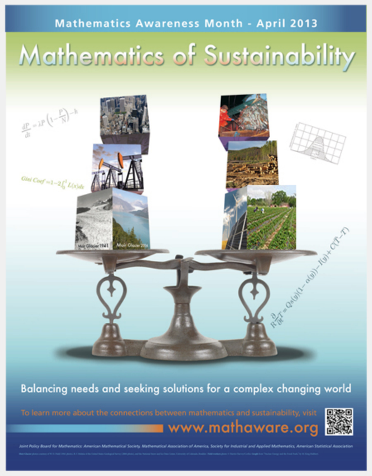

Some useful references...

- E. Mayr, What Evolution Is, (Basic Books, New York, 2001).
- A.L. Barabasi, Linked. The New Science of Networks, (Perseus Publishing, Cambridge Massachusetts, 2002.)
- F. Schweitzer, Brownian Agents and Active Particles, (Springer, Berlin, 2003)
- M.A. Nowak, Evolutionary Dynamics, Princeton Univ. Press, (2006).
- N.N. Taleb, The Black Swan: The Impact of the Highly Improbable, 2007.
- N.Bellomo, Modelling Complex Living Systems. A Kinetic Theory and tochastic Game Approach, (Birkhauser-Springer, Boston, 2008).
- K. Sigmund, The Calculus of Selfishness, Princeton Univ. Press, (2011).

Do Living, hence complex, systems exhibit common features?

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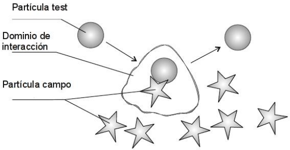
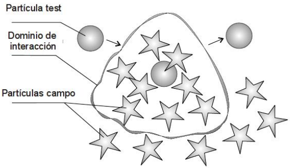
10 selected common features of complex living systems

1. Ability to express a strategy: Living entities are capable to develop specific strategies related to their organization ability depending on the state and entities in their surrounding environment. These can be expressed without the application of any principle imposed by the outer environment.
2. The said ability is not the same for all entities. Indeed, heterogeneous behaviors characterize a great part of living systems. Namely, the characteristics of interacting entities can even differ from an entity to another belonging to the same structure.
3. Large number of components: Complexity in living systems is induced by a large number of variables, which are needed to describe their overall state. Therefore, the number of equations needed for the modeling approach may be too large to be practically treated.

4. Interactions: Interactions are nonlinearly additive and involve immediate neighbors, but in some cases also distant particles, as living systems have the ability to communicate and may possibly choose different observation paths. In some cases, the topological distribution of a fixed number of neighbors can play a prominent role in the development of the strategy and interactions.



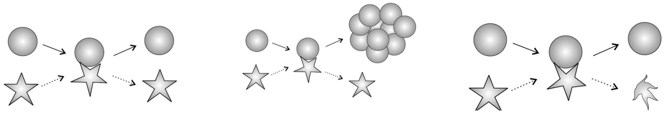
Short range interactions



Long range interactions

5. Stochastic games: Living entities at each interaction play a game with an output that is technically related to their strategy often related to surviving and adaptation ability, namely to a personal search of fitness. The output of the game is not generally deterministic even when a causality principle is identified.
6. Learning ability: Living systems have the ability to learn from past experience. Therefore their strategic ability and the characteristics of interactions among living entities evolve in time.
7. Multiscale aspects: The study of complex living systems always needs a multiscale approach. For instance in biology, the dynamics of a cell at the molecular (genetic) level determines the ability of cells to express specific functions.

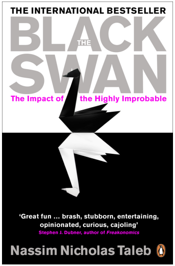
8. Darwinian selection and time as a key variable: All living systems are evolutionary. For instance birth processes can generate individuals more fitted to the environment, who in turn generate new individuals again more fitted to the outer environment.



conservative int. proliferative int. destructive int.

9. Emerging behaviors: Large living systems show collective emerging behaviors that are not directly related to the dynamics of a few interacting entities. Generally, emerging behaviors are reproduced only at a qualitative level.
10. Large deviations: The observation of living systems should focus on the emerging behaviors that appear in non-equilibrium conditions. Very similar input conditions reproduce, in several cases, the qualitative behaviors. However, large deviations can be observed corresponding to small changes in the input conditions.

What is the black swan?

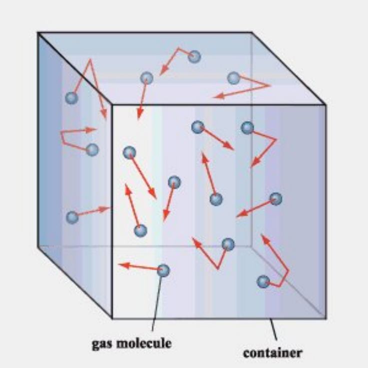


"A Black Swan is a highly improbable event with three principal characteristics: It is unpredictable; it carries a massive impact; and, after the fact, we concoct an explanation that makes it appear less random, and more predictable, than it was."

Since it is very difficult to predict directly the onset of a black swan, it is useful looking for the presence of early signals.

Outline

Kinetic theory of active particles



Hallmarks of the kinetic theory of active particles

- The overall system is subdivided into *functional subsystems* constituted by entities, called *active particles*, whose individual state is called *activity*;
- The state of each functional subsystem is defined by a suitable, time dependent, distribution function over the activity variable;
- Interactions are modeled by games, more precisely stochastic games, where the state of the interacting particles and the output of the interactions are known in probability;
- Interactions are delocalized and nonlinearly additive;
- The evolution of the distribution function is obtained by a balance of particles within elementary volumes of the space of the microscopic states, where the dynamics of inflow and outflow of particles is related to interactions at the microscopic scale.

Some applications

- There are plenty of references for applications of this framework to model complex living systems
  - Crowd dynamics and evacuation problems [Bellomo, Bellouquid, DK (2013)], [Agnelli, Colasuonno, DK (2015)]
  - Onset and evolution of criminality [Bellomo, Colasuonno, DK, Soler (2015)]
  - Vehicular traffic dynamics [Fermo and Tosin (2013)]
  - Immune competition, tumour growth, antibiotic resistance [Bellouquid, De Angelis, DK (2013)], [DK, Sánchez Sansó (2016)].
  - Migration phenomena [DK (2013)]
  - Opinion formation, learning phenomena [Ajmone, Bellomo and Gibelli (2016)], [DK (2014)].

KTAP: Formalidades

Cada partícula tiene un estado microscópico $\mathbf{w}$:

$$\mathbf{w} = (\mathbf{x}, \mathbf{v}, u) \in D_{\mathbf{w}} = D_{\mathbf{x}} \times D_{\mathbf{v}} \times D_u$$

donde

- $\mathbf{x} \in D_{\mathbf{x}}$ es el estado microscópico geométrico (por ejemplo, la posición espacial)
- $\mathbf{v} \in D_{\mathbf{v}}$ es el estado microscópico mecánico (por ejemplo, la velocidad)
- $u \in D_u$ es la actividad

KTAP: Formalidades

La descripción del estado global del sistema está dada por una función:

$$f : [0,T] \times D_{\mathbf{w}} \to [0,\infty)$$

llamada función de distribución generalizada.

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$f(t, \mathbf{w})d\mathbf{w}$ denota el número de partículas activas cuyo estado al tiempo t se encuentra en $[\mathbf{w}, \mathbf{w} + d\mathbf{w}]$. Por lo tanto

$$n_{\Lambda}(t) = \int_{\Lambda \times D_{\mathbf{v}} \times D_u} f(t, \mathbf{x}, \mathbf{v}, u) d\mathbf{x}\, d\mathbf{v}\, du$$

denota el número de partículas en $\Lambda \subset D_{\mathbf{x}}$ al tiempo t.

Desde el punto de vista de la teoría de la medida, la existencia de la función f puede garantizarse a través del Teorema de Radon-Nikodym. En efecto, si buscamos una función completamente aditiva $\phi(A)(t)$ que permita "contar" el número de partículas activas al tiempo t en cada subconjunto $A \subseteq D_{\mathbf{w}}$, dicho teorema asegura la existencia de una función integrable y no negativa f tal que $\phi(A)(t) = \int_A f(t, \mathbf{w}) d\mathbf{w}$.

KTAP: Formalidades

Conocer f permite calcular cantidades macroscópicas:

- tamaño de la población en el tiempo t en la posición $\mathbf{x}$

$$n[f](t,\mathbf{x}) = \int_{D_{\mathbf{v}} \times D_u} f(t,\mathbf{x},\mathbf{v},u) d\mathbf{v}\, du$$

- tamaño total de la población en el tiempo t

$$N[f](t) = \int_{D_{\mathbf{x}} \times D_{\mathbf{v}} \times D_u} f(t,\mathbf{x},\mathbf{v},u) d\mathbf{x}\, d\mathbf{v}\, du$$

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$N[f](t)$ no necesariamente es constante puesto que podría haber nacimientos y/o muertes, migraciones, etc.

KTAP: Formalidades

Si el número de partículas activas es constante entonces f puede normalizarse y se puede pensar como una densidad de probabilidad.

Modelando las interacciones microscópicas

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$$\eta(\mathbf{w}_*, \mathbf{w}^*)$$

  es la frecuencia de encuentros entre una partícula con estado $\mathbf{w}_*$ y una con estado $\mathbf{w}^*$.

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- Densidad de probabilidad de transición:

$$\varphi(\mathbf{w}_*, \mathbf{w}^*; \mathbf{w}) : D_{\mathbf{w}} \times D_{\mathbf{w}} \times D_{\mathbf{w}} \to \mathbb{R}_+$$

  es la probabilidad de que una partícula con estado $\mathbf{w}_*$, al interactuar con otra de estado $\mathbf{w}^*$, pase a un estado $\mathbf{w}$.

Modelando las interacciones microscópicas

Se debe satisfacer que

$$\int_{D_{\mathbf{w}}} \varphi(\mathbf{w}_*, \mathbf{w}^*; \mathbf{w}) d\mathbf{w} = 1, \quad \forall \mathbf{w}_*, \mathbf{w}^*$$

Modelando las interacciones microscópicas

Las cantidades η y φ permiten calcular flujos entrantes y salientes en el estado $\mathbf{w}$ cuando ocurren encuentros conservativos:

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- Flujo entrante C^+

$$C^+[f](t,\mathbf{w}) = \int_{D_{\mathbf{w}}\times D_{\mathbf{w}}} \eta(\mathbf{w}_*,\mathbf{w}^*)\varphi(\mathbf{w}_*,\mathbf{w}^*;\mathbf{w})f(t,\mathbf{w}_*)f(t,\mathbf{w}^*)d\mathbf{w}_*\,d\mathbf{w}^*$$

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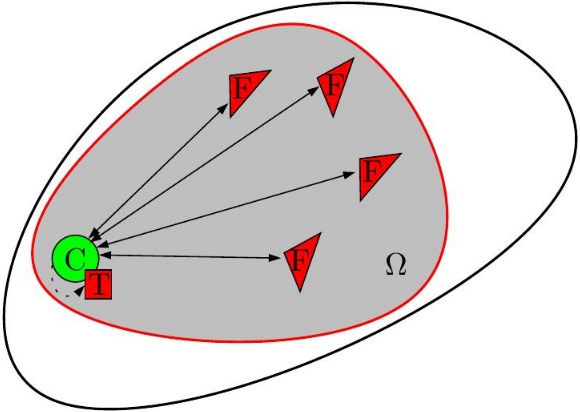
- Flujo saliente C^-

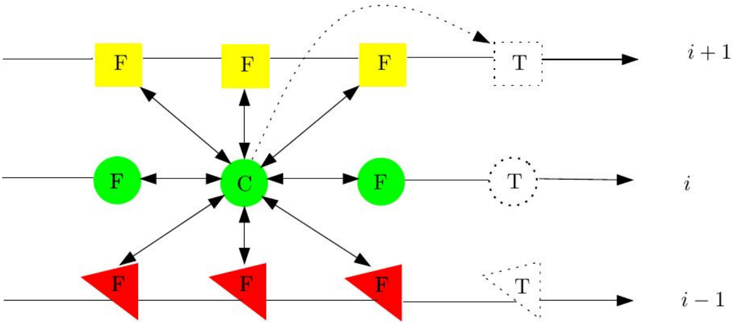
$$C^-[f](t,\mathbf{w}) = f(t,\mathbf{w}) \int_{D_\mathbf{w}} \eta(\mathbf{w},\mathbf{w}^*) f(t,\mathbf{w}^*)\, d\mathbf{w}^*$$

Modeling interactions



Active particles interact with other particles in their action domain

Modeling interactions



Active particles during proliferation move from one functional subsystem to the other through pathways.

Modelando las interacciones microscópicas

Cuando tenemos interacciones no conservativas (proliferativas y/o destructivas) se define:

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- $\nu(\mathbf{w}, \mathbf{w}^*)$ es la tasa de destrucción en el estado $\mathbf{w}$ debido a la interacción de una partícula con estado $\mathbf{w}^*$

Modelando las interacciones microscópicas

De este modo se pueden calcular cantidades macroscópicas debido a eventos proliferativos y destructivos

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- Proliferación

$$P[f](t,\mathbf{w}) = \int_{D_{\mathbf{w}}\times D_{\mathbf{w}}} \eta(\mathbf{w}_*,\mathbf{w}^*)\mu(\mathbf{w}_*,\mathbf{w}^*;\mathbf{w})f(t,\mathbf{w}_*)f(t,\mathbf{w}^*)d\mathbf{w}_*d\mathbf{w}^*$$

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- Destrucción

$$D[f](t,\mathbf{w}) = f(t,\mathbf{w}) \int_{D_{\mathbf{w}}} \eta(\mathbf{w},\mathbf{w}^*)\nu(\mathbf{w},\mathbf{w}^*) f(t,\mathbf{w}^*)\, d\mathbf{w}^*$$

Mathematical structures

$$\boxed{\begin{array}{l}\text{Variation rate of}\\ \text{the number of}\\ \text{active particles}\end{array}} = \boxed{\begin{array}{l}\text{Inlet flux rate}\\ \text{caused by}\\ \text{conservative interactions}\end{array}} - \boxed{\begin{array}{l}\text{Outlet flux rate}\\ \text{caused by}\\ \text{conservative interactio}\end{array}}$$

$$+ \boxed{\begin{array}{l}\text{Net flux of}\\ \text{active particles}\\ \text{caused by proliferative/}\\ \text{destructive interactions}\end{array}} + \boxed{\begin{array}{l}\text{Net flux from/to an}\\ \text{external agent}\end{array}}$$

Ecuaciones generales

La expresión anterior quedaría formulada de la siguiente manera:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f = C^{+}[f] - C^{-}[f] + P[f] - D[f] + E$$

término de transporte:
balance neto de partículas en el
elemento de volumen
del espacio de estados microscópicos
debido al transporte

término de interacción:
balance neto de partículas en el
elemento de volumen
del espacio de estados microscópicos
debido a las interacciones

Outline

Casos particulares

Para cada aplicación se puede particularizar la estructura general.

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Para cada aplicación se puede particularizar la estructura general. Ejemplos:

- Modelos que no dependen de las variables geométricas/mecánicas (dinámica social, formación de opiniones)
- Modelos con variable geométrica/mecánica 1D (tráfico vehicular)
- Modelos con variable geométrica/mecánica 2D (dinámica de multitudes)
- Modelos con variable geométrica/mecánica 3D (dinámica de swarms)
- Fenómenos de proliferación/destrucción (sistemas biológicos)
- Modelos donde pueden discretizarse las variables (las ecuaciones se transforman en discretas)

¿Por qué el enfoque cinético?

* **Microscópico**: Los peatones se identifican individualmente por su posición $\mathbf{x} = \mathbf{x}(t)$ y su velocidad $\mathbf{v} = \mathbf{v}(t)$ $\rightarrow$ Grandes sistemas de ODE's. [Helbing et al.: "Social force model", 1995]

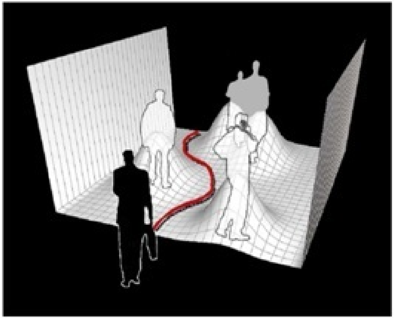
* **Macroscópico**: La multitud es vista como un continuo, y su estado se describe mediante cantidades macroscópicas promedio (densidad, momento, energía), que son variables espacio-temporales $\rightarrow$ Sistemas de PDE's [Hughes: a first order model, 2002]

∗ La distancia entre los peatones puede ser pequeña o grande, la relación entre la longitud de "camino libre promedio" y la escala de longitud representativa (número de Knudsen) puede tomar un amplio rango de valores dentro del mismo dominio.

∗ El uso de la escala macroscópica resulta complicado debido a la no validez de la hipótesis de continuidad en algunas zonas del dominio.

∗ El estudio de sistemas complejos vivientes requiere un **enfoque de tipo multiescala**.

* **Mesoscópico** (cinético): El estado microscópico de los peatones es identificado por su posición y su velocidad pero el sistema se representa a través de una función de distribución sobre dicho estado.



[J.P. Agnelli, F. Colasuonno, D.K., *A kinetic theory approach to the dynamics of crowd evacuation from bounded domains*, Math. Models Methods Appl. Sci., 25(1) (2015)]

- **Qué**: Evacuación de peatones de una habitación con una o más salidas.
- **Por qué**:

eVACUATE

A holistic, scenario-independent,
situation-awareness and guidance system
For sustaining the Active Evacuation Route for large crowds

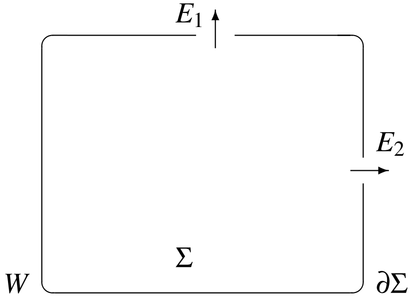
[J.P. Agnelli, F. Colasuonno, D.K., *A kinetic theory approach to the dynamics of crowd evacuation from bounded domains*, Math. Models Methods Appl. Sci., 25(1) (2015)]

- **Qué**: Evacuación de peatones de una habitación con una o más salidas.
- **Cómo**: Desarrollo del modelo cinético [N. Bellomo, A. Bellouquid, D.K., *From the Micro-scale to Collective Crowd Dynamics*, SIAM Multiscale Model. Simul. (2013)] para incluir
  - * interacciones con paredes

  - * flujo a través de salidas

Representación del sistema

- Los peatones se mueven en un dominio acotado $\Sigma \subset \mathbb{R}^2$ (que asumimos convexo).
- La frontera incluye una zona de salida $E \subset \partial\Sigma$, E que podría ser la unión disjunta de conjuntos.
- $W = \partial\Sigma \setminus E$ representa la pared.



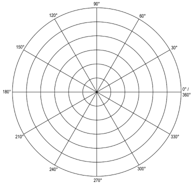
Descripción del sistema

- Los peatones son las **partículas activas**
- Estado Microscópico: (características híbridas continuas-discretas):
  - Posición: $\mathbf{x} = (x, y)$
  - Velocidad : $\mathbf{v} = (v, \theta)$
    - cambios en la dirección de velocidad θ: probabilísticos
    - cambios en el módulo de velocidad v: determinísticos

Tratamiento de la velocidad

- La dirección θ toma valores en el conjunto discreto:

$$I_\theta = \left\{ \theta_i = \frac{i-1}{n} 2\pi \, : \, i = 1, \ldots, n \right\}$$



- Los cambios en la dirección θ se modelizan desde un punto de vista probabilístico.
- El modulo de velocidad v se modeliza como una variable determinística continua que depende de efectos macroscópicos (nivel de congestión).

Función de distribución generalizada

- La función de distribución generalizada toma la forma

$$f(t,\mathbf{x},\theta) = \sum_{i=1}^{n} f_i(t,\mathbf{x})\delta(\theta-\theta_i), \quad f_i(t,\mathbf{x}) = f(t,\mathbf{x},\theta_i)$$

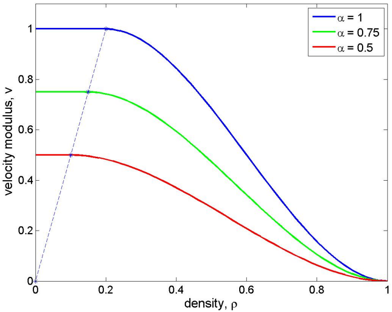
$f_i(t,\mathbf{x})d\mathbf{x}$ = número de peatones que, al tiempo t, se encuentran en el rectángulo infinitesimal $[x, x+dx] \times [y, y+dy]$ y se mueven con dirección θ_i

- Densidad local: $\rho(t,\mathbf{x}) = \sum_{i=1}^{n} f_i(t,\mathbf{x})$

Módulo de velocidad

El módulo de velocidad depende de

1. nivel de congestión (percibido)
2. la calidad del ambiente, descripta mediante un parámetro $\alpha \in [0, 1]$



In the free flow zone ($\rho \leq \rho_c(\alpha) = \alpha/5$) pedestrians move with the maximal speed $v_m(\alpha) = \alpha$ allowed by the environment. In the slowdown zone ($\rho > \rho_c(\alpha)$) pedestrians have a velocity modulus which is heuristically modeled by the 3rd order polynomial joining the points $(\rho_c(\alpha), v_m(\alpha))$ and $(1, 0)$ and having horizontal tangent in such points.

Dinámica de interacción

Tendremos en cuenta los siguientes efectos:

1. Efectos geométricos
   - Tendencia a alcanzar la salida
   - Tendencia a evitar el choque con las paredes
2. Efectos de “congestión”
   - Tendencia a moverse hacia zonas menos congestionadas
   - Tendencia a seguir el “stream”

   * ε cercano a 0 $\rightarrow$ búsqueda de áreas menos congestionadas.
   * ε cercano a 1 $\rightarrow$ tendencia pura a seguir el stream.

Modelo matemático

$$
\begin{aligned}
\partial_t f_i(t,\mathbf{x}) + \mathrm{div}_{\mathbf{x}}(\mathbf{v}_i[\rho](t,\mathbf{x}) f_i(t,\mathbf{x})) &= \mathcal{J}_i^G[f](t,\mathbf{x}) + \mathcal{J}_i^P[f](t,\mathbf{x}) \\
&= \mu[\rho(t,\mathbf{x})]\left(\sum_{h=1}^{n} \mathcal{A}_h(i) f_h(t,\mathbf{x}) - f_i(t,\mathbf{x})\right) \\
&\quad + \eta[\rho(t,\mathbf{x})]\left(\sum_{h,k=1}^{n} \mathcal{B}_{hk}(i)[\rho] f_h(t,\mathbf{x}) f_k(t,\mathbf{x}) - f_i(t,\mathbf{x})\rho(t,\mathbf{x})\right)
\end{aligned}
$$

Case-studies

Seleccionamos algunos casos específicos con el objeto de analizar la **influencia sobre el tiempo de evacuación** de

1. el **tamaño de la salida**
2. la **distribución inicial**
3. el **parámetro** ε

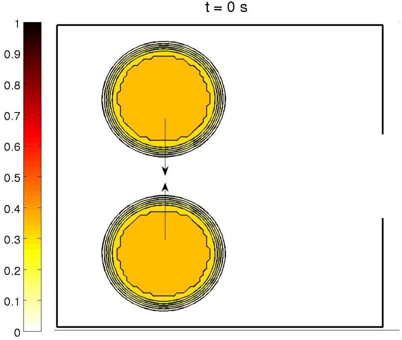
Case-studies: parámetros

En todas las simulaciones consideramos:

- Σ dominio cuadrado de lado $L = 10\,m$
- 8 direcciones de velocidad diferentes en $I_\theta = \left\{ \frac{i-1}{8} 2\pi : i = 1, \ldots, 8 \right\}$
- Calidad del ambiente $\alpha = 1$
- Módulo de velocidad referido a $V_M = 2\,m/s$
- Densidad referida a $\rho_M = 7\,ped/m^2$
- Tiempo de referencia $T_M = 13\,s$
- Tasas de interacción $\mu = 1 - \rho$ and $\eta = \rho$

The influence of exit size

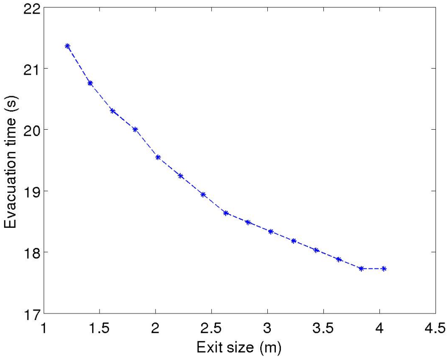
- Approximately 45 pedestrians are distributed into two circular clusters with opposite directions θ_3 and θ_7.
- Different sizes of the exit zone chosen in the range from $1{,}25\,m$ to $4\,m$.



The influence of exit size

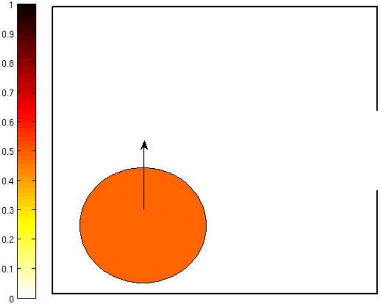
The influence of exit size

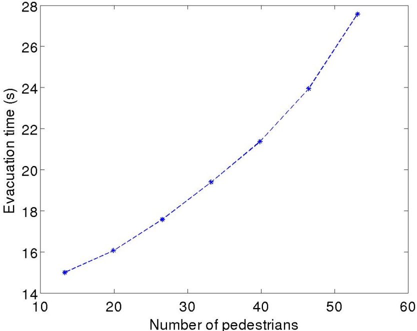
- Evacuation time increases with decreasing size of the exit.
- For sufficiently large sizes, evacuation time does not change significantly.



The influence of the initial distribution

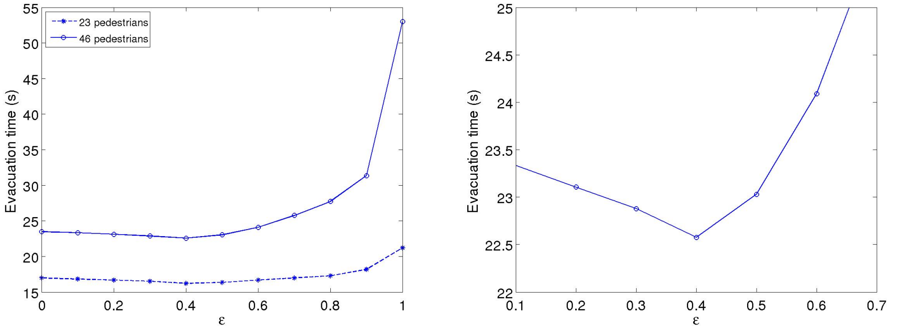
- Fixed area occupied by a crowd moving with direction θ_3.
- Different initial constant densities ρ, varying in the interval $[0.2, 0.8]$ (number of people ranging from 15 to 55).
- Door size $2\,m$.





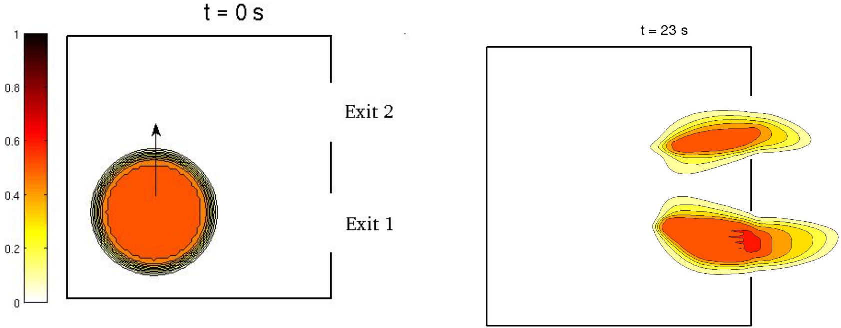
The role of the parameter ε

- Two different initial conditions (same area, different density).
- The optimal value of ε is approximately 0,4.
- $\varepsilon \approx 0 \rightarrow$ longer path to reach the exit, while $\varepsilon \approx 1 \rightarrow$ higher levels of congestion and reduction of the velocity modulus.



Room with two exits

- Ability of the model to let pedestrians decide which is the most convenient way to the exit, by taking into account not only the proximity to each door, but also the less crowded path.
- Two doors of size 2,2 m and approximately 55 pedestrians.



Room with two exits

Braess paradox in the real world: Cheonggyecheon river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Environment

Heart and soul of the city

The demolition of a vast motorway through the centre of South Korea's capital and the restoration of a river and park in its place proves that mega-cities can be changed for the better. John Vidal reports

John Vidal

Wednesday 1 November 2006 15.14 GMT

One year ago this month, several million people headed to a park in the centre of Seoul, the capital of South Korea and seventh largest city in the world. They didn't go for a rock festival, a football match or a political gathering, but mostly to just marvel at the surroundings, to get some fresh air and to paddle in the river that runs through it.

Modeling the onset and dynamics of criminality

[N. Bellomo, F. Colasuonno, D.K., J. Soler *From systems theory of sociology to modeling the onset and evolution of criminality*, Networks and Heterogeneous Media, 10(3) (2015)]

Modeling the onset and dynamics of criminality

- **What**: Modeling the onset and dynamics of criminality.

SAFECITI

- **Why**:

esri

Griffin

Functional subsystems, representation and structure

Functional subsystem	Micro-state
$i = 1$, citizens	$u \in D_1$, wealth
$i = 2$, criminals	$u \in D_2$, criminal ability
$i = 3$, detectives	$u \in D_3$, experience/prestige

Microscopic variable for each functional subsystem.

Functional subsystems, representation and structure

The representation of the system is delivered by the distribution functions

$$f_i : [0,T) \times D_i \to \mathbb{R}_0^+, \qquad i = 1,2,3, \tag{1}$$

$f_i(t,u)\,du$ denotes, under suitable integrability conditions, the number of active particles of the functional subsystem i whose state, at time t, is in the interval $[u, u+du]$. Therefore the *size* of group i is

$$n_i(t) = \int_{D_i} f_i(t,u)\,du, \quad i = 1,2,3, \tag{2}$$

At each time $t \in [0,T)$, the *total size* of the population is given by

$$N(t) = \sum_{i=1}^{3} n_i(t) \approx N_0,$$

By normalizing with respect to N_0, f_i defines the fraction of individuals belonging to a certain FS at time t.

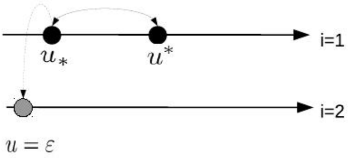
Functional subsystems, representation and structure

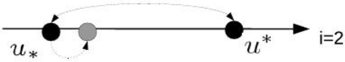
The balance of particles in the elementary volume of the space of micro-states leads to the following structure:

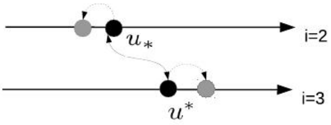
$$\partial_t f_i(t,u) = J_i[\mathbf{f}](t,u) =$$
$$= \sum_{h,k=1}^{3} \int_{D_h} \int_{D_k} \eta_{hk}(u_*,u^*) \mathcal{B}_{hk}^i(u_* \to u | u_*, u^*) f_h(t,u_*) f_k(t,u^*) \, du_* \, du^*$$
$$- f_i(t,u) \sum_{k=1}^{3} \int_{D_k} \eta_{ik}(u,u^*) f_k(t,u^*) \, du^*$$
$$+ \int_{D_i} \mu_i(u_*,\mathbb{E}_i) \mathcal{M}_i(u_* \to u | u_*, \mathbb{E}_i) f_i(t,u_*) du_* - \mu_i(u,\mathbb{E}_i) f_i(t,u) \quad (3)$$

for $i = 1,2,3$, and where the square brackets are used to denote dependence on the whole set of distribution functions $\mathbf{f} = \{f_i\}$, which is simply related, in the models proposed in this paper, to the mean value $\mathbb{E}_i$.

Modeling transitions







Parameters involved in the model

Parameters	
α_T	Susceptibility of citizens to become criminals
α_B	Susceptibility of criminals to reach back the state of normal citizens
β	Learning dynamics among criminals
γ	Motivation/efficacy of security forces to catch criminals
λ	Learning dynamics among detectives

Some selected case studies

Case 1 - Role of the mean wealth

Decreasing mean wealth of the society	$\Longrightarrow$	Increasing number of criminals Increasing criminal ability

Case 2 - Role of the shape of wealth distribution

Poor society	Equal distribution	$\Longrightarrow$	Slow growth in the number of criminals
	Unequal distribution	$\Longrightarrow$	Fast growth in the number of criminals
Rich society	Equal distribution	$\Longrightarrow$	Fast decrease in the number of criminals
	Unequal distribution	$\Longrightarrow$	Slow decrease in the number of criminals

Some selected case studies

Case 3 - Role of the number of detective

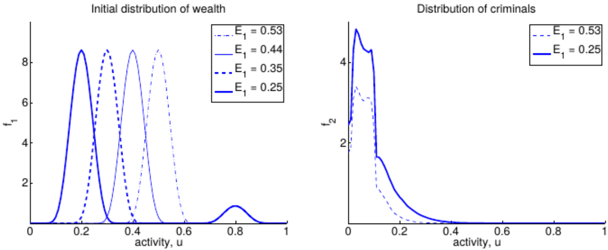
Large number of detectives	$\Longrightarrow$	Decreasing number of criminals

Case 4 - Role of parameters α_T and γ

Low susceptibility to criminality	$\Longrightarrow$	Number of criminals under control
	$\Longrightarrow$	Criminal ability under control
Increasing ability of detectives	$\Longrightarrow$	Decreasing number of criminals (with small sensitivity)
	$\Longrightarrow$	Decreasing criminal ability

Some selected case studies

Simulations - Case 1

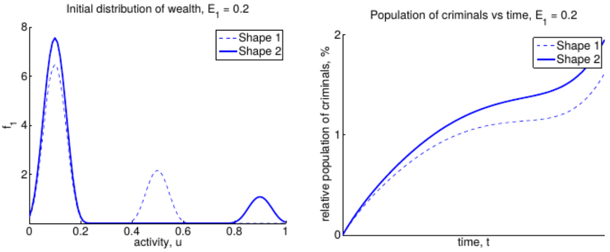


(a) (b)

(a) Initial wealth distributions used for the simulations, corresponding to different mean wealth values. All of them consist in a fixed small rich cluster and a large poorer cluster centered in different points of the activity domain. (b) Large time distri- bution of criminals for two of the selected mean wealth values.

Some selected case studies

Simulations - Case 2

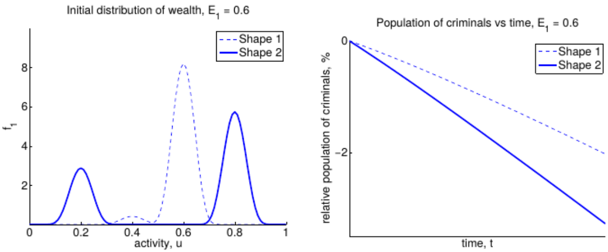


(a) (b)

(a) Two wealth distributions for a poor society with $\mathbb{E}_1 = 0{,}2$ lead to different curves for the relative growth of the population of criminals (b), where the most unequal distribution generates a larger increase.

Some selected case studies

Simulations - Case 2



(c) (d)

(c) Shows two wealth distributions for a rich society with $\mathbb{E}_1 = 0{,}6$ that generate, for the same set of model parameters, a reduction in the number of criminals (d).

Some selected case studies

Simulations - Case 3

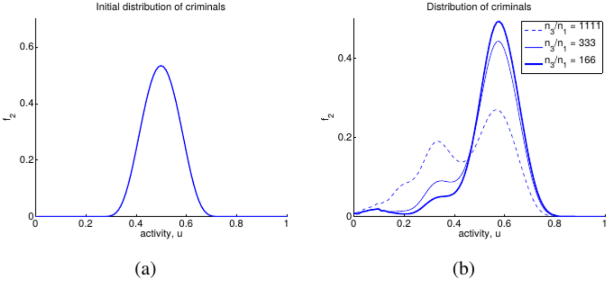


Figure 4: (a) Initial distribution of criminals. (b) Large time distribution of criminals for different number of security agents per 100,000 people.

Outline

Looking ahead... interdiscipline is needed!

- Empirical data
- Empirical data on microscopic behaviours
- Social dynamics (diffusion of types of behaviours)
- Multiscale problems
- Computational problems

Looking ahead

- Detailed analysis of bifurcation problems and asymptotic behaviors
- Interactions of different dynamics, e.g. welfare policy and criminality dynamics
- Learning collective behaviors
- Dynamics on networks
- Control problems
- Look ahead to a systems sociology approach

Gracias!