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Mixed integer programming model for synchronizing night urban bus services in Santiago City

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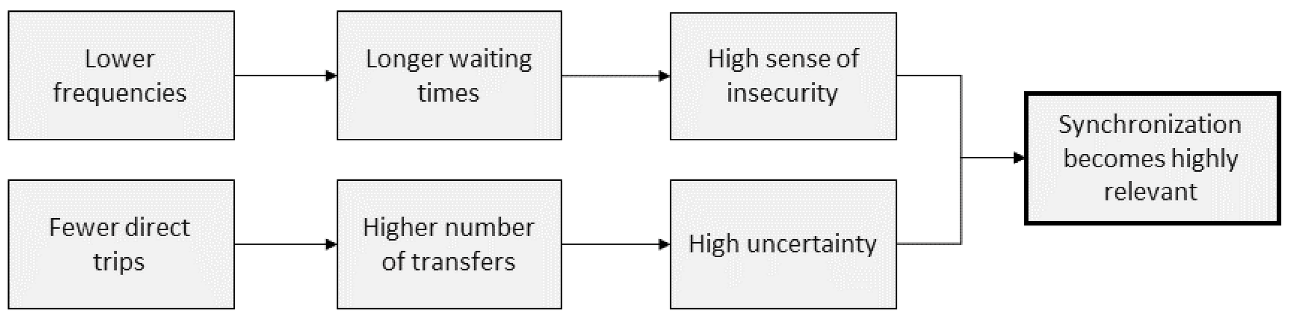
Bus Synchronization Timetabling Problem

The problem addressed in this work is the following

- Given a network of night urban bus services, the goal is to maximize the number of encounters of buses belonging to different lines that are able to perform a synchronized operation of passengers' transfers at the bus stops where that operation is possible to define a fixed schedule, satisfying systems conditions.
- Trips that sincronize arrive within a time window of allowable waiting time.
- Trips may hold at certain bus stops where such an operation is allowed.
- Maximum dwelling time per trip and bus stop capacity are addressed.
- A border limit constraint is proposed to deal with the transition between day shift and night shift.

Relevance



Decision variables

$$Y^{ij}_{pqb} \begin{cases} 1, \text{ if the arrivals of trip p of line i and trip q of line j at node b are separated by a time that is within the required waiting time limit.} \\ 0, \text{ otherwise.} \end{cases}$$

X^i departure time of the first trip of line i, $X^i \in \left[0, h^i\right]$

Z^i_b dwelling time of line i at transfer node b, $Z^i_b \in \left[0, L^i_b\right]$

S^i_b cumulative dwelling times of line i before its arrival at transfer node b

BTP Model including dwelling times at transfer stops

$$F_{BTP} = max \sum_{i \in I} \sum_{j \in J(i)} \sum_{b \in B_{ij}} \sum_{p=1}^{f_i} \sum_{q=1}^{f_j} Y^{ij}_{pqb}$$

$$X^i \leq h^i \quad \forall i \in I \tag{1}$$

$$T - h^i \leq X^i + \left(f^i - 1\right) \cdot h^i \leq T \quad \forall i \in I \tag{2}$$

$$(X^j + t^j_b + (q-1) \cdot h^j + S^j_b + Z^j_b) - (X^i + t^i_b + (p-1) \cdot h^i + S^i_b) \geq \underline{W}_b \cdot Y^{ij}_{pqb} - \underline{M}^{ij}_{pqb} \cdot (1 - Y^{ij}_{pqb})$$
$$\forall i \in I, j \in J(i), p = 1..f^i, q = 1..f^j, b \in B^{ij} \tag{3}$$

$$(X^j + t^j_b + (q-1) \cdot h^j + S^j_b + Z^j_b) - (X^i + t^i_b + (p-1) \cdot h^i + S^i_b) \leq \overline{W}_b \cdot Y^{ij}_{pqb} + \overline{M}^{ij}_{pqb} \cdot (1 - Y^{ij}_{pqb})$$
$$\forall i \in I, j \in J(i), p = 1..f^i, q = 1..f^j, b \in B^{ij} \tag{4}$$

$$S^i_{b'} = \sum_{b \in E^i : t^i_{b'} > t^i_b} Z^i_b \quad \forall i \in I, b' \in \Omega_i \tag{5}$$

New valid inequalities

$$\exists i \in I, \quad X_i = 0 \tag{6}$$

$$\begin{aligned} &if \quad (0 + t^j_b + (q-1) \cdot h^j + 0) - (h^i + t^i_b + (p-1) \cdot h^i + L^b_i \cdot O^i_b) > 0 \\ &or \quad (0 + t^i_b + (p-1) \cdot h^i + 0) - (h^j + t^j_b + (q-1) \cdot h^j + L^b_j \cdot O^j_b) > 0 \\ &\qquad then \quad Y^{ij}_{pqb} = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} &Y^{ij}_{pqb} = Y^{ij}_{p+k,q+m,b} \\ &Such\ that\ m, k \in \mathbb{N}\ where\ m \cdot h^j = k \cdot h^i = LCM(h^i, h^j) < T \end{aligned} \tag{8}$$

$$\sum_{q=q'}^{q'+\lfloor L^b_i / h^j \rfloor} Y^{ij}_{pqb} \leq 1 + (Z^i_b / h_j) \tag{9}$$

$$(1 - Y^{ij}_{p,q+1,b}) \geq (1 - Y^{ij}_{pqb}) \tag{10}$$

Experimental data

- The optimization was perform over 20 services (both ways each) and 30 bustops (8 of them with dwelling time allowed).
- The length of planning horizon (T) is 239 minutes.
- The travel times from depots of lines to nodes (t^i_b) are in the range of 0 to 98 minutes.
- The minimum and maximum separation time between synchronized trips arrivals at nodes ($\underline{W}_b$, $\overline{W}_b$) are 5 and 10 minutes, respectively.
- The maximum dwelling time allowed (L) at any node is 3 minutes.
- The headway of services 210, 401N,301 were set in 10 minutes. Headway of others services were set in 30 minutes.
- Each bus stop has a capacity of 8 vehicles.
- For this intance, border condition for each line was determined as half of its headway.

Variables	2,618,056 (10,944)
Constraints	832,673 (20,002)
Solve time (sec)	3264

All computations were coded in AMPL using the solver CPLEX 12.6 on a computer with CPU Intel Core i5, 2.20 GHz with 12 GB of RAM.

Results

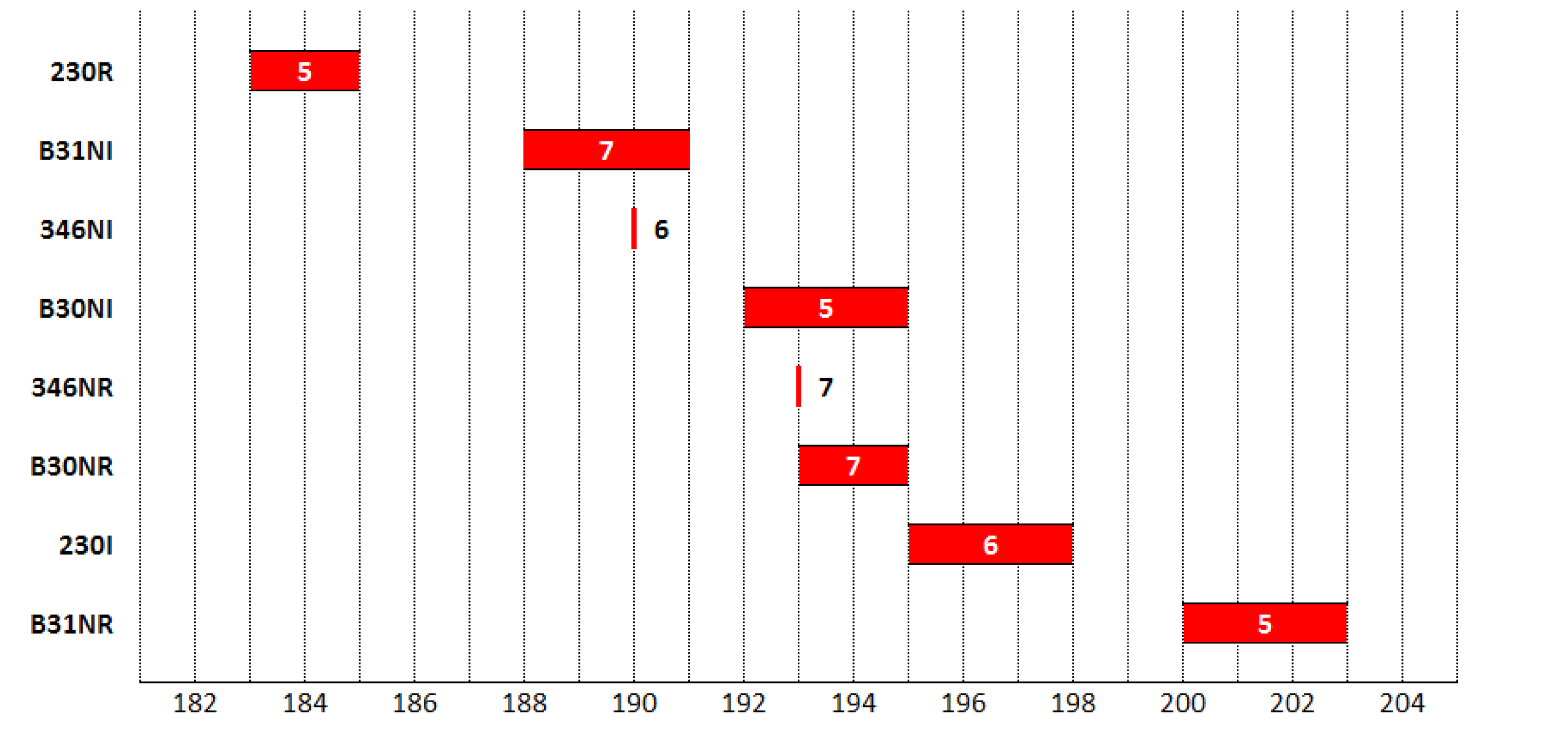
4652 synchronizations achieved between trips of lines at transfer nodes where the departure times (in minutes) of their first trips are:

104R	0	426I	14
104I	10	426R	13
107I	4	506I	5
107R	4	506R	6
112NI	1	513I	15
112NR	12	513R	12
201I	15	516I	1
201R	1	516R	15
210I	5	B02NI	15
210R	3	B02NR	3
230I	6	B30NI	12
230R	8	B30NR	12
301I	3	B31NI	7
301R	4	B31NR	5
303I	14	F28NI	9
303R	14	F28NR	7
346NI	2	F30NR	4
346NR	12	F30NI	0
405I	2	401NR	1
405R	2	401NI	0

Holding times of lines at transfer nodes are:

	Rejas	Bellavista	Departamental	Militar	Irarrazabal	Moneda	StaLucia	PlazaItalia
107R	3	0	3	0	0	0	0	0
112NI	0	0	0	2	0	0	0	0
112NR	0	0	0	3	0	0	0	0
201I	0	0	2	0	0	3	0	0
230I	0	0	0	0	0	0	3	0
230R	0	0	0	0	0	0	2	0
303I	0	0	0	0	0	3	0	0
303R	0	0	0	0	0	0	0	3
346NI	0	0	0	0	0	3	0	0
405I	2	0	0	0	0	1	0	0
405R	3	0	0	0	0	0	0	1
426I	0	0	0	2	0	3	0	3
426R	0	0	0	3	0	2	0	1
506I	0	0	0	0	1	0	0	0

The trips arriving between minute 135 and 165 of the planning horizon at bus stop Metro Irrarazabal are:



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